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Q2- Mathematics

Topic - Problem based on homogeneous equation

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Problem: Solve $2x + 5y + 2z = 0$

Solution: The given eqⁿ is

$$2x + 5y + 2z = 0 \quad (1)$$

It can be written as

$$2 \frac{\partial^2 z}{\partial x^2} + 5 \frac{\partial^2 z}{\partial x \partial y} + 2 \frac{\partial^2 z}{\partial y^2} = 0$$

$$\text{i.e. } (2D^2 + 5DD' + 2D'^2)z = 0$$

The auxiliary equation is

$$2m^2 + 5m + 2 = 0$$

$$\Rightarrow 2m^2 + 4m + m + 2 = 0$$

$$\Rightarrow 2m(m+2) + 1(m+2) = 0$$

$$\Rightarrow (m+2)(2m+1) = 0$$

$$\Rightarrow m+2 = 0 \quad \text{or} \quad 2m+1 = 0$$

$$\Rightarrow m = -2 \quad \text{or} \quad m = -\frac{1}{2}$$

Hence the solution is $z = f\left(y - \frac{1}{2}x\right) + \psi(y-2x)$
 $= \phi(2y-x) + \psi(y-2x)$

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D2- Mathematics

problem (2)

$$\text{Solve } \frac{\partial^3 z}{\partial x^3} - 3 \frac{\partial^3 z}{\partial x^2 \partial y} + 2 \frac{\partial^3 z}{\partial x \partial y^2} = 0$$

Solution: The equation can be

written as

$$(D^3 - 3D^2D' + 2DD'^2)z = 0$$

The auxiliary eqn is $m^3 - 3m^2 + 2m = 0$

$$\Rightarrow m(m^2 - 3m + 2) = 0$$

$$\Rightarrow m(m^2 - 2m - m + 2) = 0$$

$$\Rightarrow m[(m^2 - 2m) - (m - 2)] = 0$$

$$\Rightarrow m[m(m - 2) - 1(m - 2)] = 0$$

$$\Rightarrow m(m - 2)(m - 1) = 0$$

$$\Rightarrow m = 0, 2, 1$$

Hence solution is

$$z = f_1(y) + f_2(y+x) + f_3(y+2x)$$

$$\text{problem: 3 - solve } (D^3 + 4D^2D' + 4DD'^2)z = 0$$

Solution: The auxiliary eqn is

$$m^3 + 4m^2 + 4m = 0$$

$$\Rightarrow m\{m^2 + 4m + 4\} = 0$$

$$\Rightarrow m(m+2)^2 = 0 \Rightarrow m = 0, 2, 2$$

Hence the solution is $z = \phi(y) + f_1(y+x) + f_2(y+2x)$